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A PROTOTYPE SELECTION SYSTEM
FOR THE OPTIMAL CHOICE
OF AN INSTRUMENT PACKAGE

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SECTION 1

SUMMARY

General

Technical Operations, Incorporated (Tech/Ops), has developed for the Electronics Research Center of the National Aeronautics and Space Administration (NASA) a prototype selection aid for use in selecting instrument packages to be flown aboard space vehicles.

In addition, a detailed analysis of the present selection process, a quantitative formulation of the selection process, and some suggested benefit calculation and optimization techniques are presented.

Purpose

The prototype selection system was developed as an aid to long or short range project and program planning. It is a quantitative device which is used to systematize a selection process. The prototype selection aid, which utilizes a set of computer programs, sequences and formulates a quantifying system for selection. The quantifying system enables the structure of the selection process objectively. The system is not rigid, however, as it allows alternative selection structures.

The prototype system was contrived to demonstrate the potential applicability and value of such a system as an aid to project and program planning; as a systematic, quantitative approach to instrument package selection; and as a flexible aid which would allow alternative selection structures.

Approach

At some time well before the flight date, selection committees are presented with a greater number of candidate instruments than the vehicle can handle, because of weight, power, cost, or crew time constraints.

If a full scale selection system were available, it would be used as follows. The committee members vote for each instrument by attaching benefit values to each instrument for each of a number of categories, such as scientific worth, technological feasibility, etc. Each instrument will end up with a total benefit value, which is the weighted votes of the committee members, with the use of a computer program.

Given each instrument's benefit value and resource (such as weight, power, etc.) requirements and the total constraint for each resource, a program (based upon dynamic programming) chooses the combination of instruments which will be of maximum total benefit value.

An additional program gives the committee members the option of trying alternate assignments for trade-off analysis. The committee may also wish to revote or reassign benefits to candidate instruments and repeat the optimization and alternate assignment programs.

SECTION 2

INTRODUCTION

Background

NASA plans far in advance for its activities for the exploration of space. For budgetary and project development reasons, the planning has become long range. Because of this long-range planning, project instruments have had to be selected well in advance of the launch date.

In more recent times, the number of proposed instruments usually exceeded the limitations of the already-defined space vehicle. This led to the exclusion of some and the inclusion of others, based upon prevailing constraints. It is expected that in the future the total number of candidate instruments for automated spacecraft will greatly exceed the available space vehicle capacity. Moreover, increasingly complex constraints may be imposed.

The greater number of instruments and constraints may make it difficult to examine all possible vehicle packages. For this reason, there is a need to develop quantitative techniques with which NASA can systematize and optimize the selection of the instrument package for a particular flight or a program.

Tech/Ops has developed a prototype selection aid and an example for the above problem. In addition, a detailed analysis of the present selection process, a quantitative formulation of the selection process, and some suggested benefit calculation and optimization techniques are presented.

The system is designed as an aid to the planning of instrument packages for future flights or programs. The development is a systematic, mathematical approach which calculates total benefit values for each instrument, determines the optimal assignment, and explores alternate assignments.

Limitations

The prototype selection aid has certain limitations, partly due to the present computer capabilities and partly due to the depth of this study, which is essentially a feasibility analysis. The following details these limitations.

1. The mathematical algorithm used for optimization, dynamic programming, can use only linear benefit relationships for this problem. Refer to Appendix B for a more detailed explanation.
2. The product of the number of instruments times the constraint value must be less than 1400. In the prototype example, 25 instruments and a maximum constraint of 55 is handled.
3. A maximum of one constraint at one time can be dealt with.

Limitations 2 and 3 are due to program language (BASIC, a Dartmouth College invention) and storage constraints, and will be eliminated with the use of a larger computer and FORTRAN language.

Full Scale System Development

A full-scale decision making system should embody features, not available in the prototype, relating to greater ease of use and relaxation of the limitations described above. Since items 2 and 3 of the limitations of the prototype are due to the restricted computer employed, they present no major problem. Further investigation is required to develop an algorithm to handle nonlinear benefit relationships. However, many situations can be accommodated by a linear benefit relationship, and it is felt that a full scale system could be developed in parallel with a study of the nonlinear methods. Investigation of other voting procedures, use of options, methods of input of data, and improved report formats could be performed during the development of the full scale system. No major difficulty is anticipated in implementing the system, within six to nine months with two to three individuals, for use by one of the committees or for one of the flight programs.

The authors wish to express their gratitude to Professor David W. Conrath, University of Pennsylvania, for his participation in the area of decision making in the research undertaken for this contract.

SECTION 3

PROTOTYPE SELECTION SYSTEM

General

The Prototype Selection System demonstrates the types of capability that would assist NASA decision makers. The system consists essentially of three stages (see Figure 1): vote taking; prototype computer programs, which are the VOTING, OPTO, and CHECK programs; and the analysis of results. The stages are described below and an example follows:

Stage 1: Vote Taking

General. --The vote taking proceeds in three steps (see Figure 2): instrument characteristics are supplied to the voters for evaluation of instruments; the voters vote on the merits of the instruments; and their votes are tabulated for use in the prototype computer programs.

Instrument Characteristics. -- The voters are provided with information involving the various instruments. It includes instrument descriptions (weight, cost, power requirement, and function); developer ideas (whether the instruments will be ready in time and if they can be improved); the backgrounds of the various instruments (if they have been tried and tested under the actual conditions of the flight, and, if so, if there is any additional benefit to be derived from repeated inclusion); and expert opinions, both pro and con, on the above.

Voting. --The vote taking itself involves two sets of evaluation by each voter. First, several categories are presented to the voters. These might include ones as scientific value of the instrument, potential follow-up research resulting from inclusion of the instrument, technological value of the instrument, and reliability of the instrument. Each voter then evaluates the relative significance of these categories in the light of the overall purpose of the mission and rates each category between 0 and 10 where higher numerical rating reflects greater significance.

Second, each voter evaluates each instrument in each of the above categories, again rating the instruments from 0 to 10 in each category with 10 corresponding to the greatest possible value of an instrument in that category.

Vote Data. --The results of the above voting are then tabulated for use in the prototype computer programs.

Stage 2: Prototype Computer Programs

General. --Three programs constitute this section. They consist of the VOTING, OPTO and CHECK programs described below.

Voting Program. --Utilization of the VOTING program requires that the results of the vote data be fed into the computer. Thereupon, the VOTING program calculates the benefit values to be ascribed to each instrument. Refer to Figure 3.

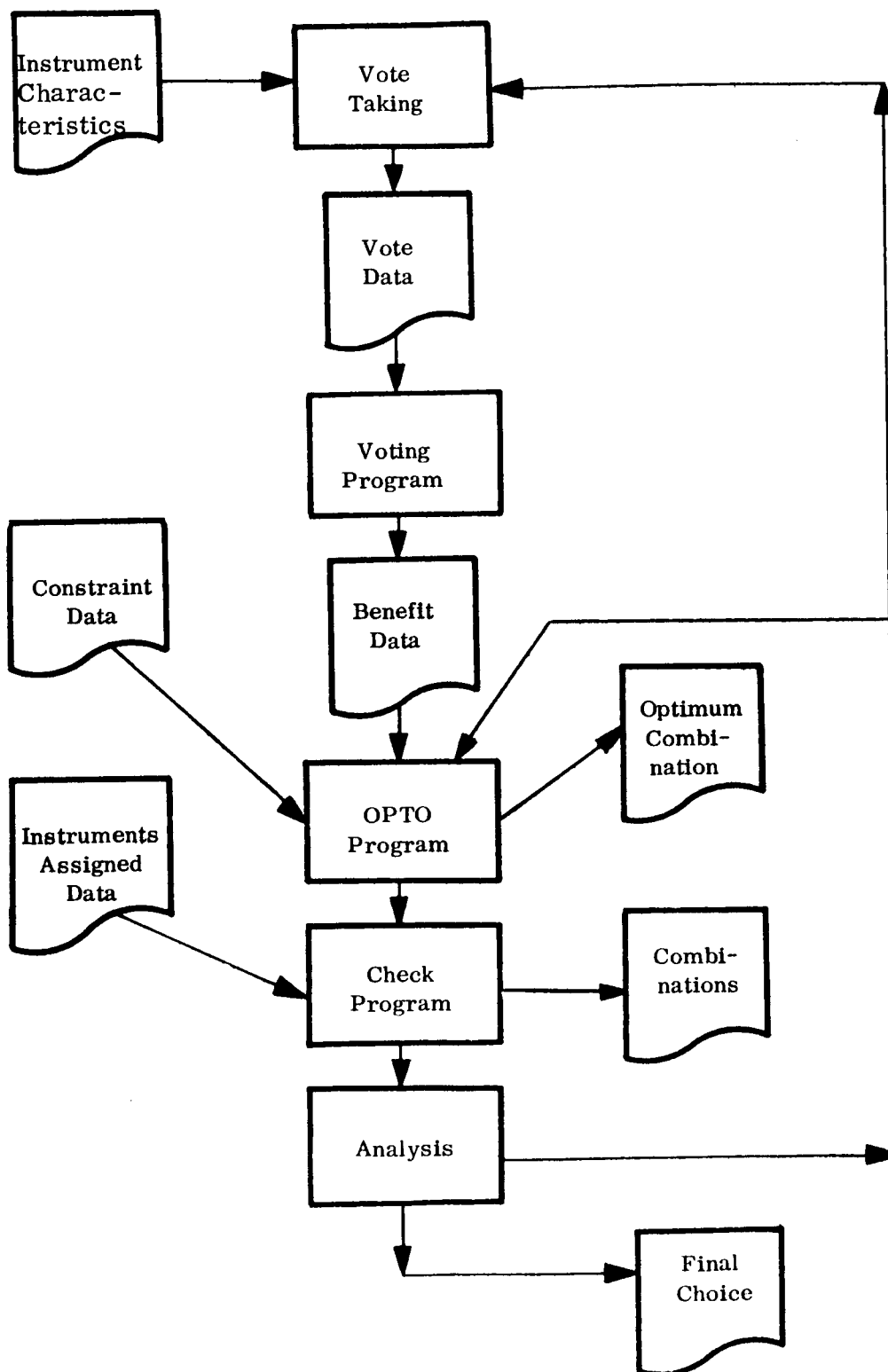


Figure 1. - Prototype Selection System

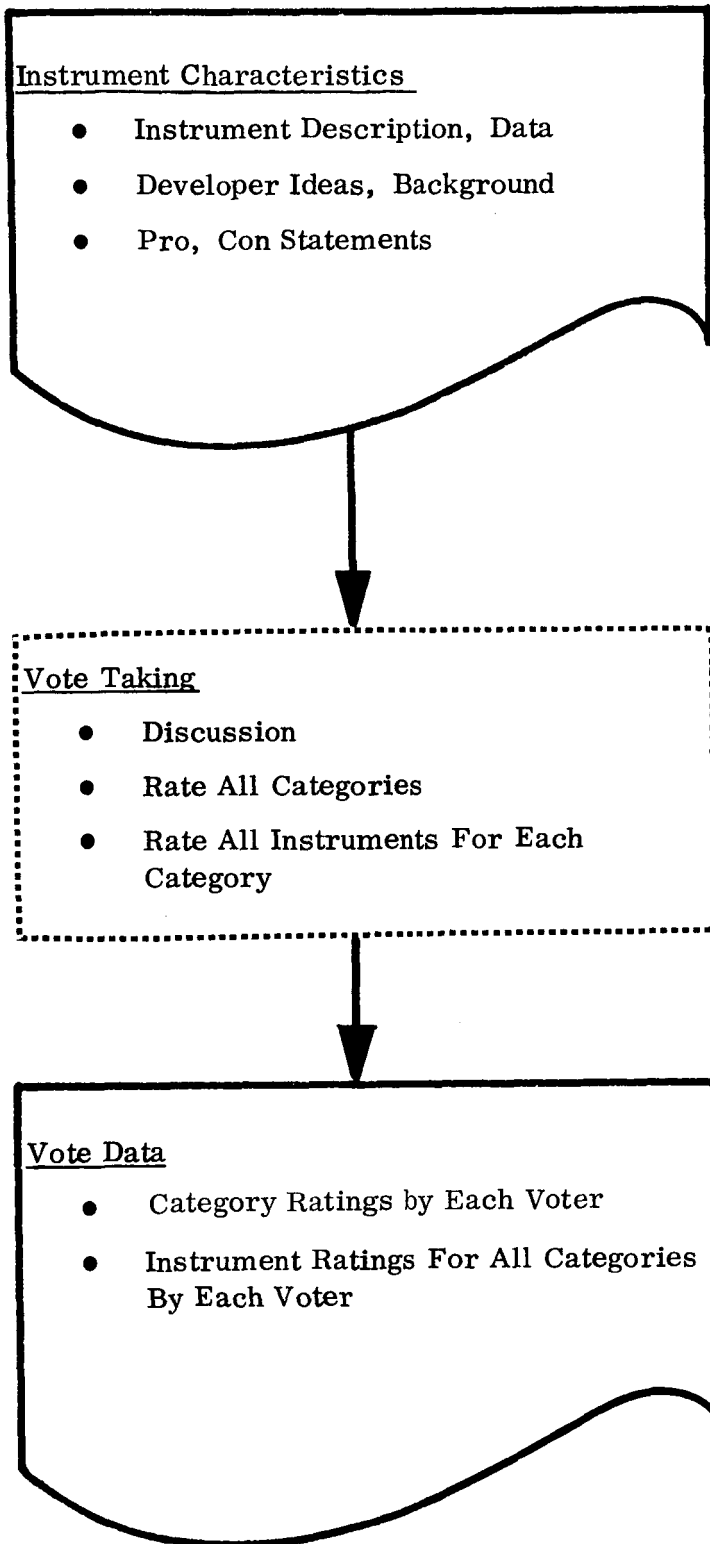


Figure 2. - Vote Taking

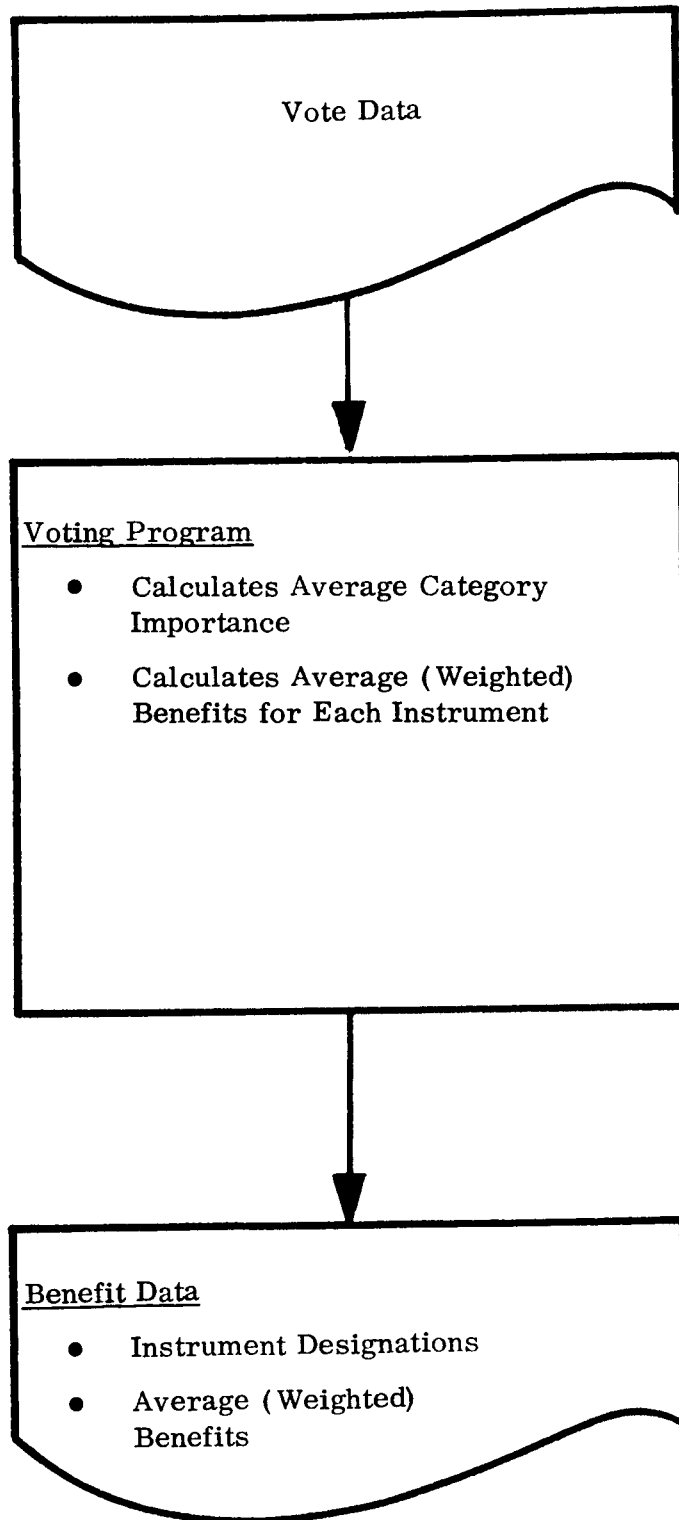


Figure 3. - VOTING Program

Opto Program. --To employ the OPTO program, the user proceeds in the following manner (see Figure 4). First, he feeds in the constraint data (which may be the weights of the instruments, their costs, or their power requirements, and the total resource constraint). Next, the user feeds the benefit data into the computer.

The OPTO program now provides the user with the optimal selection of instruments not only for the originally planned total constraint (50 in the prototype), but also for total constraints varying between 10 percent less to 10 percent more than the planned total constraint, in increments of one constraint unit.

Further, at each of these constraint values, the OPTO program computes the total resources used by the assignment of the optimal selection for this constraint as well as the benefit produced by this assignment.

Check Program. --Having been provided by the OPTO program with the optimal selection of instruments satisfying a total constraint of 50, the CHECK program allows the user to try other selections of the instruments and compare them with the optimal selection (see Figure 5). This is done by simply instructing the computer to give the value of the included instruments a "1" and the excluded instruments a "0" and reading into the computer the constraint values and the benefit values of all instruments. The CHECK program then calculates how much resource the user's selections have used and how great a benefit the user's selections have produced.

Stage 3: Analysis

Having found (through employment of OPTO) the optimal selections of instruments for varying constraints and having found (through use of CHECK) what nonoptimal selections produce, the user may now proceed to analyze his information with several considerations in mind. Refer to Figure 6.

It may be that a certain previously excluded pair or triplet of instruments when considered individually have a cumulatively smaller benefit than when considered collectively. Correspondingly, packaging them together changes their cumulative constraint (possibly increasing it). In such a case, the user could re-apply OPTO, this time with the new packaged set of instruments viewed as a single instrument.

It might also occur that several of the instruments have almost identical quotients of benefit value to constraint used. Thus, it might happen that as the total available resources changes slightly, different experiments are included or excluded from the optimal payload. In such a case, a discussion by voters with the aim of re-evaluating these experiments is advisable. If confidence in the original evaluations persists, then, within allowable constraints, the instrument(s) producing optimal benefit would, of course, be selected. If not, reapplication of VOTING would be in order.

Whatever the course of action in the above situations, the final results will be a list of instruments to be included in the payload, the benefit produced by this set of instruments, and the resources used by them.

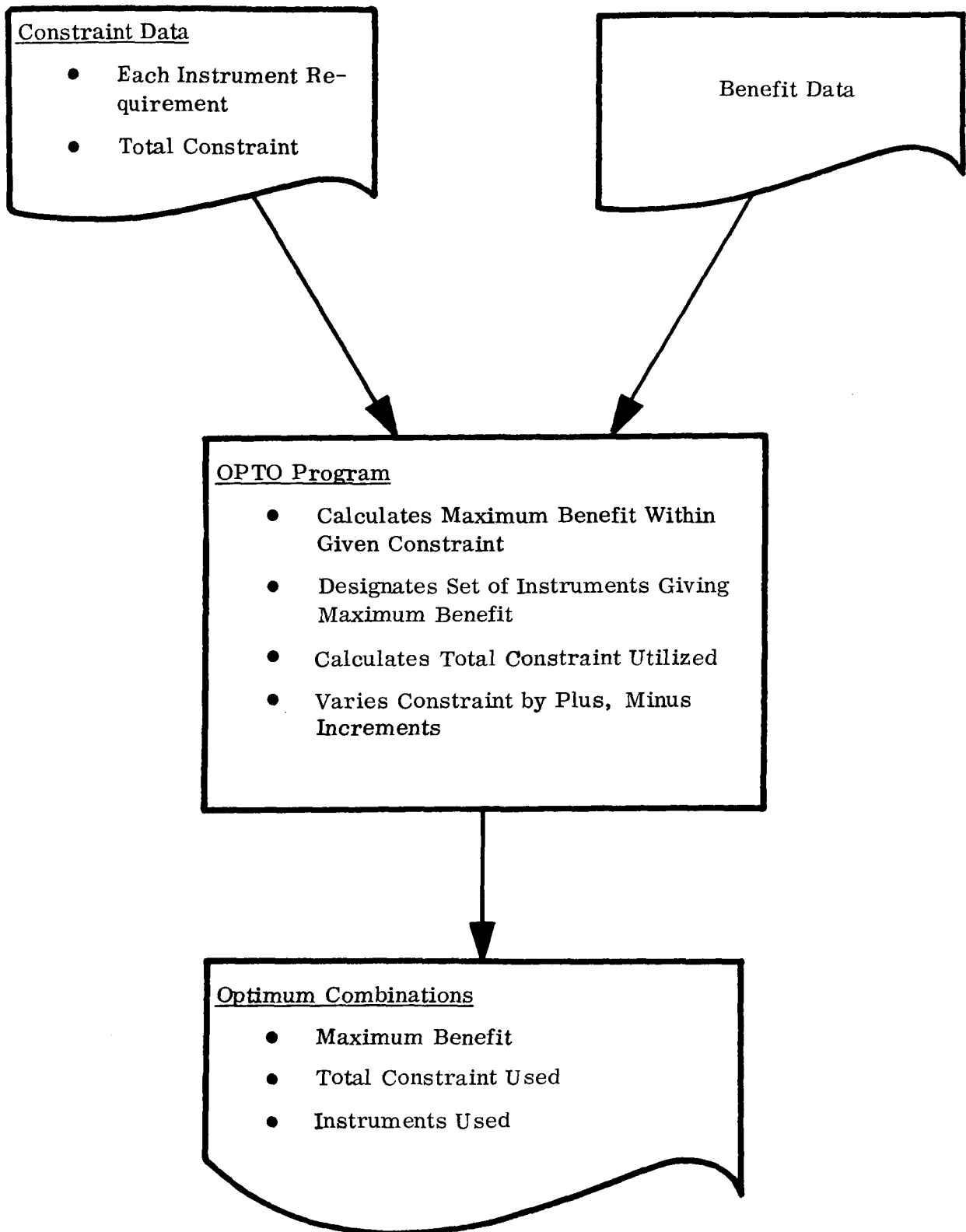


Figure 4. - OPTO Program

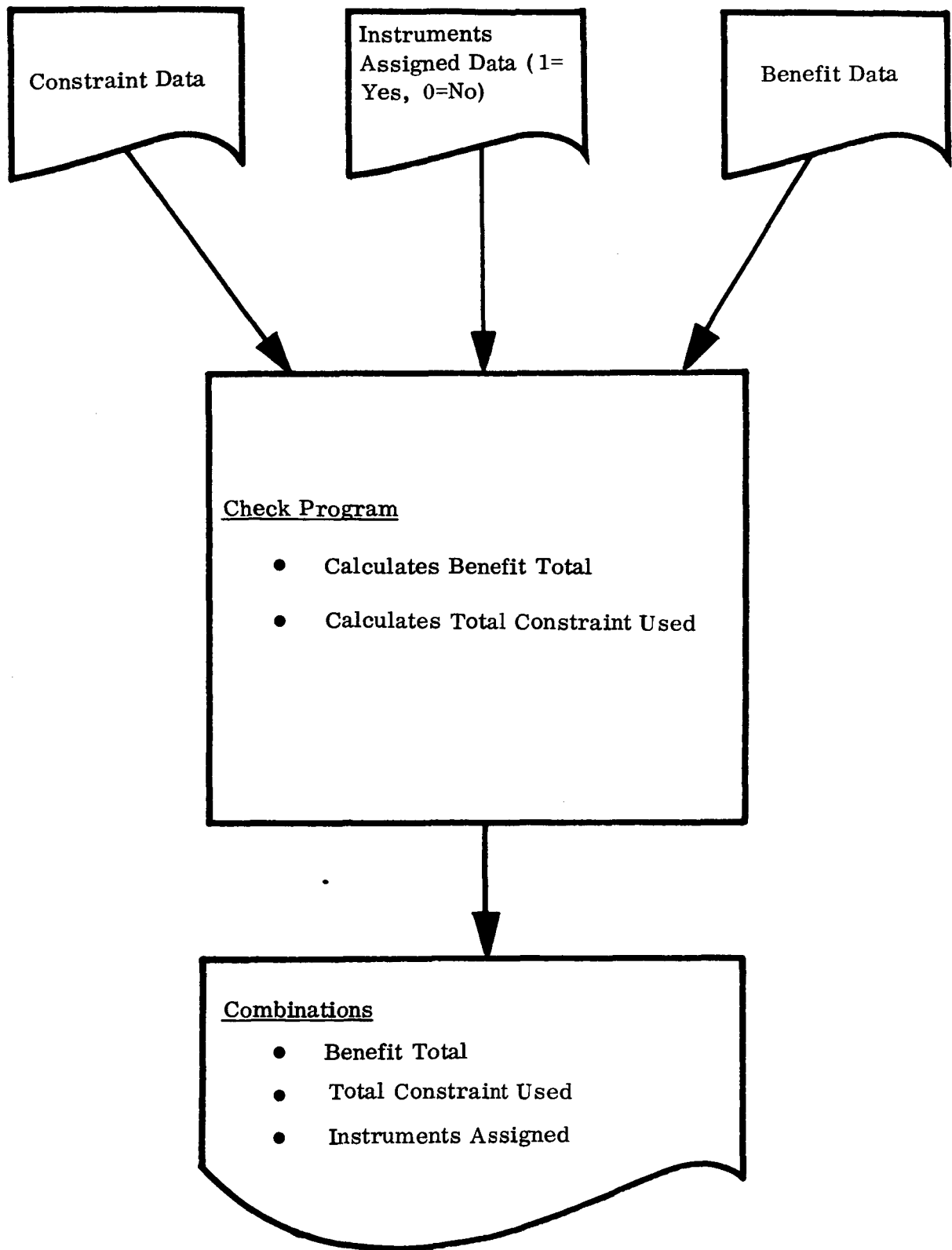


Figure 5. - CHECK Program

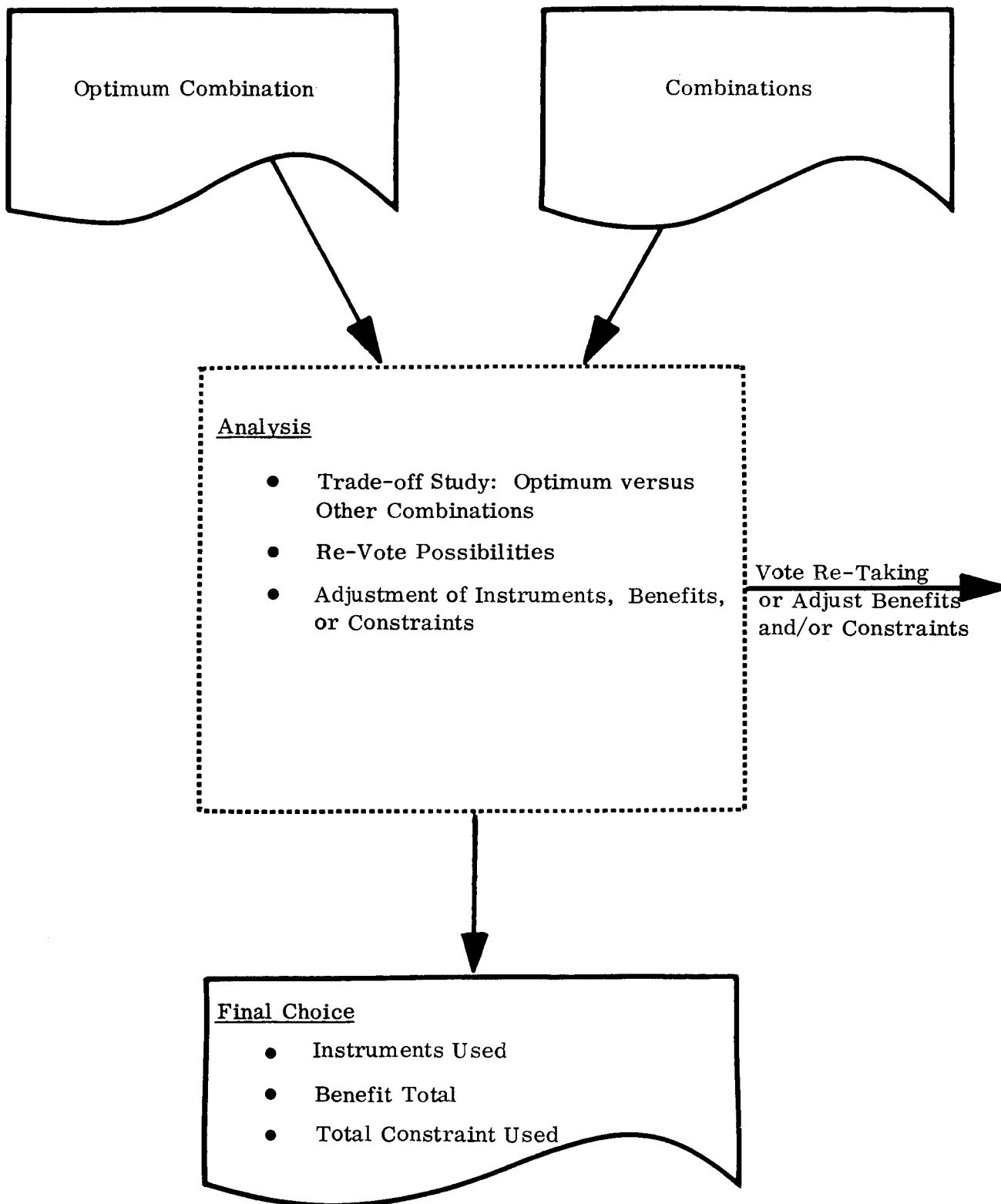


Figure 6. - Analysis

Example

Figures 7, 8 and 9 illustrate an example of an instrument package selection problem. Twenty-five candidates are proposed for inclusion in a given flight, which has a power constraint of fifty watts.

A sample voting form is shown in Figure 7. As an example, eight committee members voted for the importance of each of four categories (see Figure 8 for an explanation of the typical categories) and the value of each instrument with respect to each of the four categories. The results of the benefit calculations, which were arrived at by use of the VOTING program and the committee members' votes, are shown in column 3 of the table in Figure 9.

Given the constraint (requirement) associated with each instrument, as shown in column 2 of the table, and the benefit of each instrument (column 3), the OPTO program determined which instruments to include for the given total constraint, fifty watts, which was additionally varied by ten percent on the plus and minus sides. Column 4 indicates the instrument selection (a "1" for instrument inclusion, "0" for exclusion) and the total benefit (bottom row) associated with the instruments selected for a total constraint of 45 watts. Column 5 shows similar numbers for a total constraint of 46 watts; and so on, through 55 watts.

The shaded portion in the table indicates cases where an instrument was included or excluded in the lower total constraint but whose state of inclusion or exclusion changed when the next higher constraint limit was considered. Examination of these changing states of the instrument and of the erratic behavior of the optimal benefit numbers for changes in the total constraint limit (from 45 to 46, i. e.,) is useful for trade-off studies. For example, committee members may wish to weigh the extra benefit and instrument inclusion-exclusion changes derived against the additional cost associated with an increase in power (from 50 to 51 watts, i. e.,) if an increase in power is considered.

<u>Instrument Number</u>	<u>Categories</u>			
	1	2	3	4
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
Category Importance				

Instructions:

1. Vote 0 up to 10 (highest value) for category importance for each category.
2. Vote 0 up to 10 (highest value) for each instrument relative to each category.

Figure 7. - Sample Voting Form

<u>Category Number</u>	<u>Explanation</u>
1. Scientific Value of Instrument	The intrinsic value or benefit to mankind derived from the successful use of the instrument.
2. Follow-up Research	The additional research, related to the goal of the experiment, which results from use of the instrument utilized in the experiment.
3. Technological Value of Instrument	The technological benefit, to business or government, derived from use of the instrument.
4. Reliability of Equipment	The degree of reliability provided by previous space performances of the instrument.

Figure 8. - Sample Voting Key

Instrument			Optimal Combination										
			Constraint Restriction										
No.	Constr.	Benefit	45	46	47	48	49	50	51	52	53	54	55
1	2	164.8	1	1	1	1	1	1	1	1	1	1	1
2	4	142.2	1	1	1	1	1	1	1	1	1	1	1
3	15	139.8	0	0	0	0	0	0	0	0	0	0	0
4	6	127.3	0	0	1	1	1	0	1	1	1	1	1
5	2	124.2	1	1	1	1	1	1	1	1	1	1	1
6	14	127.3	0	0	0	0	0	0	0	0	0	0	0
7	12	136.7	0	0	0	0	0	0	0	0	0	0	0
8	6	128.9	1	1	1	1	1	1	1	1	1	1	1
9	1	128.1	1	1	1	1	1	1	1	1	1	1	1
10	2	117.2	1	1	1	1	1	1	1	1	1	1	1
11	4	112.5	1	0	0	0	0	1	1	0	0	0	0
12	3	149.2	1	1	1	1	1	1	1	1	1	1	1
13	1	136.7	1	1	1	1	1	1	1	1	1	1	1
14	2	133.6	1	1	1	1	1	1	1	1	1	1	1
15	1	123.4	1	1	1	1	1	1	1	1	1	1	1
16	3	132.0	1	1	1	1	1	1	1	1	1	1	1
17	5	113.3	0	1	0	0	0	1	0	1	1	1	1
18	3	115.6	1	1	1	1	1	1	1	1	1	1	1
19	4	115.6	1	1	1	1	1	1	1	1	1	1	1
20	6	111.7	0	0	0	0	0	0	0	0	0	0	0
21	2	122.6	1	1	1	1	1	1	1	1	1	1	1
22	3	119.5	1	1	1	1	1	1	1	1	1	1	1
23	16	121.1	0	0	0	0	0	0	0	0	0	0	0
24	2	135.2	1	1	1	1	1	1	1	1	1	1	1
25	17	118.0	0	0	0	0	0	0	0	0	0	0	0
Optimal Benefit			2201.3	2202.1	2216.1	2216.1	2216.1	2314.6	2323.6	2329.4	2329.4	2329.4	2329.4

Key: 1 = instrument included, 0 = instrument excluded.

Figure 9. - Example of Optimum Selection

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APPENDIX A

THE GENERAL PROBLEM FORMULATION

I. Introduction

The general mathematical representation of the NASA experiment - instrument selection process is discussed below. The formulation is quite general and admits of many conceivable problem situations. A particular instrument selection problem is posed and solved in Appendix D, and whose computer solution appears in Section 3.

II. The Mathematical Formulation

The set of experiments E from which the final choices are to be made can be represented as

$$E = \left\| \begin{array}{cccc} \vec{E}_{11}(t), & \vec{E}_{12}(t) & \text{----} & \vec{E}_{1M(1)}(t) \\ \vec{E}_{21}(t), & \vec{E}_{22}(t) & \text{----} & \vec{E}_{2M(2)}(t) \\ \vdots & & & \\ \vec{E}_{N1}(t), & \vec{E}_{N2}(t) & \text{----} & \vec{E}_{NM(N)}(t) \end{array} \right\| \quad (1)$$

where

N = the number of experiment disciplines (i.e., physics, astronomy, bioscience, etc.) .

$M(n)$ = the number of experiments proposed to the final selection board in discipline n .

and each element in the matrix.

$$\begin{aligned} \vec{E}_{ij}(t) &= \text{the experiment vector, representing experiment proposal } j \text{ of} & (2) \\ &\text{discipline } i, \text{ at time } t. \text{ The index } j \text{ does not represent a} \\ &\text{ranking but rather an assignment number in discipline } i. \\ &= (e_{ij1}(t), e_{ij2}(t), e_{ij3}(t), \text{----} e_{ijLL(t)}(t)) . \end{aligned}$$

where

$e_{ijk}(t) = 0$ or 1 , depending on whether instrument or equipment k is used (1) or not used (0) in experiment $\vec{E}_{ij}(t)$, as of time t .

$LL(t) =$ the total number of instruments and equipment which are used in the experiments proposed at time t .

t = the time (month, day, year) at which experiment selection is made by the final selection group.

Note that some experiments may be considered multidisciplinary, and hence, may be included in several disciplines simultaneously, so that

$$\vec{E}_{ij}(t) = \vec{E}_{qr}(t) = \vec{E}_{lm}(t) = \text{----}, \text{ etc.}$$

At the time of consideration, t , by the final selection board, each experiment j of category i and instrument or equipment k will have a probability of being developed and ready for inclusion in the payload at some specified time prior to the launch date, which probabilities are expressed as $P_{ij}(t)$ and $P_k(t)$, respectively.

The choices of experiments are constrained by weight, power, crew time, volume allotment, data transmission rate, functioning reliability, and perhaps other limitations, which can be expressed as a vector

$$\vec{L} = \vec{L} (L_1, L_2, L_3 \text{ ---- } L_p(t))$$

where

$P(t)$ = the total number of limitations (constraints)

and

$$\begin{aligned} L_{p(t)} &\geq \sum_i \sum_j \sum_k X_{ij} e_{ijk} L_{kp(t)} \\ &- \sum_l \sum_m \sum_i \sum_j \sum_k X_{ij}(t) X_{lm}(t) K_{ijlmk}(t) e_{ijk}(t) e_{lmk}(t) L_{kp(t)} \\ &- \sum_n \sum_o \sum_l \sum_m \sum_i \sum_j \sum_k X_{ij}(t) X_{lm}(t) X_{no}(t) K_{ijlmnok}(t) e_{ijk}(t) e_{lmk}(t) e_{nok}(t) L_{kp(t)} \\ &- \dots \end{aligned} \quad (3)$$

in which

$X_{ij}(t)$ = 0 or 1, depending upon whether experiment j of discipline i is chosen (1) or not chosen (0) for the payload, at time t .

$K_{ijlmk}(t)$ = the reduction or increase factor (in weight, power, crew time, etc., depending upon the constraint $L_{p(t)}$) when both experiments \vec{E}_{ij} and \vec{E}_{lm} use the same instrument k , rather than separate instruments or equipment at time t . It may be positive or negative in sign.

$K_{ijlmnok}(t)$ = the reduction or increase factor when three experiments, \vec{E}_{ij} , \vec{E}_{lm} , \vec{E}_{no} share the same instrument or equipment (k), at time t . It may be positive or negative in sign.

$L_{p(t)}$ = the quantity of the p^{th} limitation (constraint) available at time t .

$L_{kp}(t)$ = the numerical size (20 lbs, for instance) of instrument k with respect to limitation p (1500 lbs, i.e.,) at time t .

Finally, with the above experiment definitions and limitations in mind, the final selection group would like to maximize benefit (B) at some intermediate time t and/or final time T by choosing the proper combination of experiments proposed and/or developed, and is expressed as

$$\max_{i,j} B(X_{ij}(t), P_{ij}(t), P_k(t), b_{ij}(t), b_{ijkl}(t), b_{ijklmn}(t), \dots) \quad (4)$$

which choices are constrained by equation (3),

where

$b_{ij}(t)$ = the additional benefit at time t derived when experiment j of discipline i is included in the payload.

$b_{ijkl}(t)$ = the additional incremental benefit at time t derived when both experiments \bar{E}_{ij} and \bar{E}_{kl} are included, in addition to the benefit contribution made by each separately.

$b_{ijklmn}(t)$ = the additional incremental benefit at time t derived when all three experiments \bar{E}_{ij} , \bar{E}_{kl} , \bar{E}_{mn} are included, in addition to the benefit contribution made by each separately.

$p_{ij}(t)$ = the probability at time t that experiment j of category i will be developed and ready for inclusion in the payload at time T .

$p_k(t)$ = the probability at time t that instrument or equipment k will be developed and ready for inclusion in the payload at time T .

APPENDIX B

RELATIONSHIP OF THE NASA STRUCTURE TO THE GENERAL PROBLEM FORMULATION

I. Introduction

The problem formulation in Appendix B was necessarily general, for flexible application and broad useage. The way in which one arrives at a useful application of this formulation is discussed here.

II. The Relationship

The experiment proposals as of some time t , referred to in the set of experiments E , should be identified and defined by committees in the selection process. That is, experiments which are distinct and separate from other experiments in objectives (uses) or equipment/instrument usage should be enumerated by and within each discipline by the appropriate subcommittees.

Probabilities at this time of being developed and ready for payload inclusion at some specified prelaunch time should be calculated by these same groups. Value rankings or just priorities of experiments should be established, if necessary, and assigned for each discipline in accordance with schemes approved by higher level selection groups (discussed later). Also, instrument characteristics such as weight, power available or requirements, size, etc., should be logged for the instrument used for each proposed experiment.

Next, the total number of distinct instruments and equipment should be determined and enumerated, and each experiment vector \vec{E}_{ij} defined in terms of the instruments/equipment used. This should occur with initial^{ij} selection committees, where cognizance of all experiments facilitates the final selection determination.

The matrix set of experiments E , made up of the experiments \vec{E}_k should now be constructed at the highest selection level. A separate tabulation should identify multidisciplinary experiments and the disciplines for which they were simultaneously considered.

The highest selection level group then should identify and quantify the constraints, such as weight, power, space available, crew time, etc., which exist for the prospective launch or series of launches.

Likewise the highest selection level group should determine the constraint reduction factors which apply when two or more experiments utilize or share the same piece(s) of equipment or instrument(s). One of the most difficult tasks for the group is to decide whether to maximize benefit B over an entire project, program, or just one flight. Maximizing the benefit for a flight will not necessarily do the same for a project, and vice-versa. Decisions such as this are a major policy one, and probably should be decided in the highest NASA level or in congressional budget hearings. Likewise, it may be difficult to decide whether to maximize benefit periodically, (and if so, how often), only at the beginning or end of the experiment development phase, or in some cumulative time fashion (an average benefit, say).

Another difficult task to accomplish is to determine the form of the benefit function, B. If it is linear in terms of the experiment choice variables X_{ij} , it is assumed that the benefit added by inclusion of an experiment or experiments in a payload is independent of the benefits present or absent as a result of inclusion or exclusion of other experiments. In short, no gain in benefit occurs when certain combinations of experiments are included in the payload. Linearity of the benefit function B can be expressed as

$$B(t) = \sum_i \sum_j b_{ij}(t) X_{ij}(t) P_{ij}(t) \prod_k (P_k(t))^{e_{ijk}(t)} \quad (1)$$

which represents the product of yes-no choice of experiment \bar{E}_{ij} , the probability that it will be developed and ready in time, the probability that all instruments used in the experiment will be developed and ready in time, and the benefit derived from inclusion of the experiment in the payload. The formula assumes all instruments are needed for the success of the experiment. Reduction factors, representing loss in benefit when certain instruments/equipment are omitted from an experiment package, can be incorporated into the benefit coefficients b_{ij} .

If B is nonlinear with respect to the (experiment) choice variables X_{ij} , the assumption is that benefit gain occurs when certain combinations of experiments are present. Then B can be represented as

$$\begin{aligned} B(t) = & \sum_i \sum_j (b_{ij}(t) X_{ij}(t) P_{ij}(t) \prod_k (P_k(t))^{e_{ijk}(t)} \\ & + \sum_l \sum_m b_{ijlm}(t) X_{ij}(t) X_{lm}(t) P_{ij}(t) P_{lm}(t) \prod_k P_k(t)^{e_{ijk}(t) + e_{lmk}(t)} \\ & + \sum_l \sum_m \sum_n \sum_o b_{ijlmno}(t) X_{ij}(t) X_{lm}(t) X_{no}(t) P_{ij}(t) P_{lm}(t) P_{no}(t) \\ & \quad \prod_k P_k(t)^{e_{ijk}(t) + e_{lmk}(t) + e_{nok}(t) + \dots} \end{aligned} \quad (2)$$

Probably the most difficult task to accomplish by the final selection committee will be that of determining the form of the benefit coefficients $b_{ij}(t)$ for a linear benefit function, and of additional coefficients $b_{ijlm}(t)$, $b_{ijlmno}(t)$, etc., for a nonlinear (gain) benefit function. The coefficients would depend upon values assigned to or voted for an experiment's discipline, by the highest level selection group, to the experiment ranking within the discipline performance criteria categories, the number of experiments proposed for each discipline, the number and levels of the voting participants, the confidences of the voting participants in their own and others' ability to evaluate experiments under consideration and experiment benefit reduction factors for experiment output losses due to the nonavailability of instruments/equipment for inclusion in the payload. Notationally, the coefficients would appear as

$$b_{ij}(t), b_{ijkl}(t), b_{ijklmn}(t), \text{ etc.}, = f(A_{i3n}, B_{i3n}, a_{ijmnl}, n_i, v_{mn}, c_{mnop}, r_{ijk}) \quad (3)$$

where

- A_{i3n} = the value assigned by member n of the highest level selection group to discipline i .
- B_{i3n} = the value assigned by member n of the highest level selection group to category 1. These categories, such as technological value, are considered closely related to program or project goals, and hence form a backdrop against which experiments must be compared or rated against. See Appendix D for examples.
- a_{ijmnl} = the value assigned by member n of the m^{th} level group to experiment j of relative to Category 1.
- n_i = the number of proposed experiments in discipline i .
- v_{mn} = the number of voters for discipline n of level m .
- c_{mnop} = the confidence of voter p or N voters of level m in p 's ability to evaluate and vote on experiment o of discipline n .
- r_{ijk} = the benefit reduction factor on experiment j of discipline i because of the nonavailability of instrument/equipment k to the payload.

There are essentially eight possible experiment selection situations which might exist:

Situation 1. --The benefit function is linear, the constraints are linear, and only one level is considered for selection. This corresponds to independence of benefit for each candidate experiment (no gain present), no sharing of equipment by several experiments, and only the highest committee level in the selection process makes a selection decision (the other levels make only recommendations on the inclusion or exclusion of a candidate experiment).

Situation 2. --The benefit function is linear, the constraints are nonlinear, and decisions are made on only one level. This means no gain is present, but some experiments share equipment, and only the highest committee level in the selection process makes selection decisions.

Situation 3. --The benefit function is linear, the constraints are linear, and ranking and selection decisions are made at all committee levels. This situation corresponds to no gain present, no experiments share the same equipment, but decisions are made by subcommittees, the SSSC or OARTEB, and the MSFEB, successively.

Situation 4. --The benefit function is linear, the constraints nonlinear, and decisions are made successively at the three levels. This means the candidate experiments' effectiveness are independent of the inclusion of other experiments (no gain present), experiments share equipment, and decisions are made by the subcommittees, the SSSC or OARTEB, and the MSFEB, successively.

Situation 5. --The benefit function is nonlinear, the constraints are linear, and selection decisions are made at only one level. The situation corresponds to the dependence of effectiveness of some experiments upon the inclusion of others (gain present), no experiments share equipment, and selection decisions are made by the MSFEB or SSSC only.

Situation 6. --The benefit function is nonlinear, the constraints are linear, but decisions are made at all levels.

Situation 7. --The benefit function is nonlinear, the constraints are nonlinear, and decisions are made at only one level.

Situation 8. --The benefit function is nonlinear, the constraints are nonlinear, and decisions are made at all levels.

These problems can be solved quickly and effectively if there are a small number of candidate experiments (up to fifteen or so) by computing all combinations. When the number of experiments becomes larger, however, dynamic programming or pseudo-boolean methods are more efficient, practical means of solution.

From a mathematical viewpoint, the eight situations mentioned above can be reduced to two distinct problems: Situation 1 is an integer linear problem — no gain is experienced when certain combinations of experiments are chosen for payload inclusion, experiments do not share common equipment, and only one decision stage (at the highest level) occurs; the other situations represent varying combinations of linear and nonlinear aspects of an experiment selection problem, and can be represented by the most general situation, Situation 8, where gain is experienced, some experiments share equipment, and varying intensities of recommendations and decisions are made at all levels.

APPENDIX C

SOLUTION TECHNIQUES

I. Introduction

The general problem formulation was discussed in Appendix B of this section. Examination of the present NASA experiment selection procedures and proposal situation revealed the practicality and feasibility of considering the benefit function to be approximately described in a linear fashion by equation (1) of Appendix C. In addition, the probabilities of successful development of the experiment and hardware are assumed to be equal to one, the different experiment disciplines are considered to have no significant differences in values relative to the program's or project's goals, and only one selection set is made over the project planning time span. Thus, equation (1) of Appendix C becomes

$$B = \sum_i b_i x_i$$

Furthermore, each experiment is represented by a unique instrument, and only one (linear) constraint is considered. Since no gain is assumed in the benefit function B (linearity is assumed), the constraint equation becomes

$$\sum_{k=1}^n w_k \cdot x_k \leq w$$

where

w_k = the weight (cost, physical weight, power requirement, etc.,) of the k^{th} instrument.

x_k = 0 or 1, depending upon whether instrument k is included (1) or excluded (0) in the payload.

w = the total allowable weight or constraint.

A prototype selection aid, developed to solve the above problem, is discussed in this appendix and previously in Section 3.

The core of the prototype selection aid consists of benefit calculations for each candidate instrument and the optimization procedure. The benefit calculation system is composed of benefit vote taking and benefit vote averaging processes, and results in the assignment of relative values to each instrument. The optimization procedure is based upon dynamic programming theory and yields the set of instruments which cumulatively give the highest possible benefit under the given constraints. The following explains the theory behind both the instrument benefit calculation and the optimization procedure.

II. Instrument Benefit Calculations

Instrument benefit calculations b_i are arrived at by quantifying and averaging instrument and category importance votes of the selection committee members. The crux of the problem in obtaining instrument benefit values lies in quantifying member opinions, in relating the worth of one member's opinion to that of another member, and in arriving at a composite (overall) benefit value for each instrument.

In quantifying member opinions, instruments must be measured against a set of performance criteria. The following list is suggestive of a reasonable set of criteria against which candidate instruments should be measured to obtain instrument worths with respect to each criteria.

1. Reputation of the developer (scientist) — The general competence or capability of principal investigator.
2. Support of the institution — The financial, technological, and staff aid capabilities of the principal investigator's institution.
3. Scientific value of the instrument — The intrinsic value or benefit to mankind derived from the successful use of the instrument.
4. Propaganda value of the instrument — The propaganda effect on other nations and individuals upon successful completion of the experiment in which the instrument plays a key part.
5. Monetary value of the instrument — The financial benefit for the business or government community derived from use of the instrument.
6. Technological value of the instrument — The technological benefit for business or government derived from use of the instrument.
7. Follow-up research — The additional research, related to the goal of the experiment, which results from use of the instrument.
8. Existence of conceptualization — The degree to which the instrument has been concretely conceptualized and to which the theory has been validated.
9. Existence of technology — The degree to which the instrument is/will be technologically feasible to construct.
10. Reliability of the instrument — The degree of reliability provided by previous or anticipated space performances of the instrument.

Once a reasonable set of criteria has been chosen, it is necessary to quantify the relative importances of each criterion. Essentially, two methods exist: each member can be allocated a fixed quantity of votes he may distribute in any manner amongst the criteria; or each member may be allowed to vote within a given equivalent range for each criterion. The second seems preferable, and was employed in the prototype selection system since the first allows less independent judgment for each criterion and less flexibility in the total number of votes cast.

For analogous reasons, the second method can be used by members to judge each candidate's relative worth for each criterion. This approach was used for the prototype selection system. Similarly, two ways exist to determine the worth of one member's opinions versus another's: the democratic way (one man, one vote) can be employed; or relative weights can be attached to each member's votes by dint of committee position or seniority, or by a self-judged or member's averaged capability index for each member. The first seemed preferable, and was used in the prototype selection system since relative capabilities are hard to determine or vote on.

Finally, there are essentially two principal ways of combining the member's votes for criteria importance and votes for individual instruments relative to each criterion, to arrive at composite criteria importances and composite candidate instrument values relative to each criterion: the arithmetic average or the median. The arithmetic average is marginally preferable for reasons of computer use facility.

In summary, the benefit b_k for instrument k is computed as

$$b_k = \frac{\sum_{i=1}^N \sum_{j=1}^J w_{ijk} \cdot A_j}{N} ,$$

where

$$A_j = \frac{\sum_{i=1}^N v_{ij}}{N} , \text{ the average importance of criterion } j ,$$

and

w_{ijk} = the vote, from 0 to 10, of member i for instrument k 's value relative to criterion j .

v_{ij} = the vote, from 0 to 10, of member i for the relative importance of criterion j .

J = the number of criteria.

N = the number of selection committee members.

III. Optimization Technique

The optimization technique employed in the prototype selection aid system is based upon dynamic programming. The following sections describe the general mathematical approach to the optimization problem under consideration, the general solution, and the particular formulation and solution of the NASA instrument selection problem.

General Problem. -- Suppose there are n variables x_1, \dots, x_n and it is desired to maximize a function of the form

$$B(x_1, x_2, \dots, x_n) = f_1(x_1) + \dots + f_n(x_n) \quad (1)$$

where

$f_1(0) = f_2(0) = \dots = f_n(0) = 0$, and the x_k 's are limited by the conditions:

$$x_k \text{ is a non-negative integer,} \quad (2)$$

and

$$\sum_{k=1}^n a_k \cdot x_k \leq C, \quad (3)$$

where

$$a_k \geq 0 \text{ and } C \geq 0.$$

General Solution. -- Let $F(i, j)$ denote the maximum value of $B(x_1, \dots, x_i, 0, \dots, 0)$ where x_1, \dots, x_i satisfy (2) above and are subject to the condition:

$$\sum_{k=1}^i a_k x_k \leq j$$

Then $F(i, j)$ may be calculated using the recursion formula

$$F(i, j) = \supremum \{f_i(x_i) + F(i-1, j - a_i x_i) \mid a_i x_i \leq j\}$$

with $F(0, 0) = 0$.

The maximum of $B(x_1, \dots, x_n)$ is then $F(n, C)$.

Mathematical Formulation of the Prototype Instrument Selection Problem. -- Let n be the number of instruments under consideration and let x_i be the variable describing the i^{th} instrument, where $x_i = 1$ if the i^{th} instrument is included in the payload, and $x_i = 0$ if the i^{th} instrument is not included in the payload. Since the voters have already determined the benefit to be derived from each instrument's inclusion, we can see that maximizing the function

$$B(x_1, \dots, x_n) = b_1 x_1 + \dots + b_n x_n,$$

where b_i is the benefit derived from inclusion of the i^{th} instrument, will produce optimal benefit for the entire payload. It is clear that if

$$f_i(x_i) = b_i \cdot x_i$$

then $f_i(0) = 0$ for all i . Further, the payload has a total weight restriction (cost, power requirement, etc.,) so that a condition of the form

$$\sum_{k=1}^n w_k \cdot x_k \leq w,$$

where w_k is the weight of the k^{th} instrument and w is the total allowable weight, must be imposed. Thus, the prototype instrument selection problem falls into the category described in the General Problem.

Application of General Solution to Prototype Instrument Selection Problem. --Since the Prototype Instrument Selection Problem falls within the category of the General Problem, the General Solution is amenable for application. However, due to the fact that the x_i 's have a limited range of 0 or 1, the problem solution is greatly simplified. In fact, the $F(i, j)$'s have the simple form

$$F(i, j) = \left\{ \begin{array}{l} F(i-1, j), \text{ if } a_i > j \\ \text{Max. } [F(i-1, j), b_i + F(i-1, j-a_i)], \text{ if } a_i \leq j \end{array} \right\}$$

The importance of this particularly simple form of the $F(i, j)$'s will be realized if the procedure is viewed as used in the OPTO program to determine a combination of the x_i 's which will produce an optimal benefit. To determine the value of x_n , we look at whether or not $F(n, w)$ is unequal to $F(n-1, w)$. If so, then $x_n = 1$ (since the only way of changing $F(i, j)$ by increase in i is to add the i^{th} instrument and drop some other(s)); otherwise, let $x_n = 0$.

Proceeding inductively, after finding values of x_n, \dots, x_{j+1} , it is desired to find the value of x_j . Suppose k of the resource is still left. Then, if $F(j, k)$ is unequal to $F(j-1, k)$, let $x_j = 1$; otherwise, let $x_j = 0$. To find the value of x_1 , test to see if there is sufficient resource left for x_1 to be included: if so, $x_1 = 1$; otherwise, $x_1 = 0$.

APPENDIX D
COMPUTER PROGRAM LISTINGS

I. VOTING Program

VOTING

```
10 DIM V(8,4)
15 DIM A(4)
20 DIM B(32,25)
30 DIM K(32)
45 DIM Y(8)
55 DIM E(25)
60 DIM Z(8)
66 FOR I = 1 TO 8
77 FOR J = 1 TO 4
88 READ V(I,J)
98 NEXT J
99 NEXT I
111 FOR J = 1 TO 4
112 LET C = V(1,J)
122 FOR K = 2 TO 8
133 LET C = C + V(K,J)
144 NEXT K
145 LET A(J) = C/8
155 NEXT J
222 FOR N = 1 TO 32
223 FOR K = 1 TO 25
233 READ U(N,K)
244 NEXT K
245 NEXT N
333 FOR K = 1 TO 25
334 FOR J = 1 TO 8
335 FOR M = 1 TO 4
344 LET X(M) = J + (8*(M-1))
355 NEXT M
356 LET D = U(J,K)
366 FOR M = 2 TO 4
377 LET F = X(M)
378 LET D = D + U(F,K)
379 LET Y(M) = D*A(M)
380 NEXT M
381 LET Z(J) = Y(M)
382 NEXT J
383 LET G = Z(1)
384 FOR H = 2 TO 8
385 LET G = G + Z(H)
386 NEXT H
390 LET B(K) = G/8
399 NEXT K
400 GO TO 501
501 PRINT " INSTRUMENT NO.1" B(1)
```

```

502 PRINT " INSTRUMENT NO.2"$(2)
503 PRINT " INSTRUMENT NO.3"$(3)
504 PRINT " INSTRUMENT NO.4"$(4)
505 PRINT " INSTRUMENT NO.5"$(5)
506 PRINT " INSTRUMENT NO.6"$(6)
507 PRINT " INSTRUMENT NO.7"$(7)
508 PRINT " INSTRUMENT NO.8"$(8)
509 PRINT " INSTRUMENT NO.9"$(9)
510 PRINT " INSTRUMENT NO.10" $(10)
511 PRINT " INSTRUMENT NO.11" $(11)
512 PRINT " INSTRUMENT NO.12" $(12)
513 PRINT " INSTRUMENT NO.13" $(13)
514 PRINT " INSTRUMENT NO.14" $(14)
515 PRINT " INSTRUMENT NO.15" $(15)
516 PRINT " INSTRUMENT NO.16" $(16)
517 PRINT " INSTRUMENT NO.17" $(17)
518 PRINT " INSTRUMENT NO.18" $(18)
519 PRINT " INSTRUMENT NO.19" $(19)
520 PRINT " INSTRUMENT NO.20" $(20)
521 PRINT " INSTRUMENT NO.21" $(21)
522 PRINT " INSTRUMENT NO.22" $(22)
523 PRINT " INSTRUMENT NO.23" $(23)
524 PRINT " INSTRUMENT NO.24" $(24)
525 PRINT " INSTRUMENT NO.25" $(25)
901 DATA 7,3,5,6,
902 DATA 10,9,4,7,
903 DATA 6,8,6,6,
904 DATA 9,6,5,4,
905 DATA 8,8,4,9,
906 DATA 10,9,5,6,
907 DATA 9,9,3,4,
908 DATA 8,7,2,3,
909 DATA 10,9,2,1,7,7,4,3,5,9,6,4,5,
910 DATA 6,4,7,7,3,9,1,0,5,0,6,3,
911 DATA 6,8,7,7,4,3,8,9,6,4,5,7,3,
912 DATA 9,4,3,2,7,1,6,4,5,5,2,1,
913 DATA 8,8,7,6,1,4,7,3,5,1,10,10,0,
914 DATA 6,7,4,4,2,2,9,2,3,5,6,4,
915 DATA 8,7,5,3,9,8,4,6,5,3,5,10,2,
916 DATA 6,7,3,2,1,1,0,7,6,4,4,1,
917 DATA 8,9,7,2,1,1,0,3,6,4,7,4,10,
918 DATA 6,6,5,3,0,7,6,0,10,2,4,6,
919 DATA 3,7,10,4,0,5,6,2,3,1,2,1,6,
920 DATA 5,5,6,2,7,3,1,6,7,9,5,10,
921 DATA 6,7,3,2,1,9,0,10,7,7,3,2,7,
922 DATA 5,3,2,1,9,6,8,1,1,5,7,6,
923 DATA 1,7,9,8,3,3,8,5,3,2,7,6,5,
924 DATA 9,3,2,6,7,4,0,0,0,7,6,5,
925 DATA 9,5,4,7,3,5,3,7,0,1,2,7,7,
926 DATA 6,5,3,2,1,4,3,3,3,4,1,7,
927 DATA 0,5,3,8,9,6,4,5,3,2,7,5,6,
928 DATA 10,9,3,2,7,6,6,4,9,0,2,5,

```

929 DATA 10,0,5,9,3,3,4,2,7,8,7,5,6,
 930 DATA 9,0,3,0,2,7,8,0,0,4,7,6,
 931 DATA 7,7,9,2,4,3,8,3,9,5,4,3,7,
 932 DATA 0,0,9,10,3,5,4,8,0,3,2,1,
 933 DATA 9,9,3,7,8,1,0,3,7,6,3,2,1,
 934 DATA 6,6,2,1,4,3,8,9,9,7,6,7,
 935 DATA 10,3,0,5,4,3,2,0,4,7,5,3,1,
 936 DATA 3,0,5,6,5,4,7,2,5,3,0,1,
 937 DATA 5,3,3,10,4,8,6,3,2,1,0,7,8,
 938 DATA 0,4,3,3,7,5,3,8,4,5,7,9,
 939 DATA 3,7,4,3,0,9,0,3,5,7,3,3,9,
 940 DATA 9,0,5,6,4,3,1,1,4,5,7,6,
 941 DATA 4,3,9,0,4,6,5,7,9,3,2,4,10,
 942 DATA 6,10,3,3,2,6,9,3,6,7,6,5,
 943 DATA 0,5,6,10,9,1,3,2,4,3,7,8,5,
 944 DATA 9,6,3,0,6,4,7,8,2,4,3,6,
 945 DATA 3,7,9,2,0,6,10,5,5,4,6,9,3,
 946 DATA 4,7,8,8,3,0,5,9,6,6,3,1,
 947 DATA 9,1,0,8,4,3,9,5,7,4,3,7,3,
 948 DATA 3,2,0,9,5,6,3,5,8,9,4,2,
 949 DATA 3,4,0,10,10,4,0,7,9,10,1,4,3,
 950 DATA 2,5,7,3,4,7,8,2,2,4,3,4,
 951 DATA 6,0,4,9,8,5,10,3,10,4,3,7,9,
 952 DATA 0,5,6,1,7,1,4,6,3,2,4,9,
 953 DATA 9,8,4,3,7,8,5,2,8,3,2,5,4,
 954 DATA 5,0,4,3,2,1,4,3,2,7,8,0,
 955 DATA 5,4,10,3,7,8,6,1,2,7,6,5,5,
 956 DATA 7,9,3,8,5,6,4,2,3,4,3,6,
 957 DATA 8,0,10,5,4,3,4,7,3,3,2,5,4,
 958 DATA 2,9,10,6,3,4,2,1,1,0,10,1,
 959 DATA 3,7,10,2,4,3,9,6,0,1,7,9,1,
 960 DATA 6,4,8,10,4,3,2,8,8,9,5,4,
 961 DATA 10,0,4,6,8,3,10,9,4,5,4,10,9,
 962 DATA 4,7,8,8,4,3,1,9,1,4,6,8,
 963 DATA 9,4,7,8,10,4,6,8,9,2,2,1,4,
 964 DATA 4,8,9,5,3,3,2,9,7,8,4,5,
 965 DATA 8,5,0,9,0,10,9,8,6,7,5,6,1,
 966 DATA 6,0,5,4,4,8,7,7,9,10,4,2,
 967 DATA 2,4,5,0,1,7,8,9,1,10,3,1,2,
 968 DATA 0,5,7,5,2,9,8,7,5,4,8,7,
 969 DATA 6,10,9,0,5,4,3,2,1,4,7,4,3,
 970 DATA 4,10,3,9,7,8,0,10,7,8,5,1,
 971 DATA 3,9,1,4,7,10,9,2,3,5,1,6,7,
 972 DATA 4,8,5,6,3,0,1,3,0,3,2,1,
 999 END

II. OPTO Program

OPTO

```
40 DIM A(25)
50 DIM X(25)
55 DIM F(26,56)
56 DIM K(25)
60 LET F(0,0) = 0
61 FOR N = 1 TO 25
62 READ A(N)
63 READ K(N)
64 NEXT N
80 FOR N = 1 TO 25
100 LET F(N,0) = 0
160 FOR K = 1 TO 55
180 LET F(0,K) = 0
200 IF A(N) <= K THEN 245
220 LET D = -1
240 GO TO 273
245 LET Z = N-1
250 LET W = K - A(N)
260 LET D = B(N) + F(Z,W)
278 LET Z = N-1
280 IF F(Z,K) < D THEN 340
300 LET F(N,K) = F(Z,K)
320 GO TO 345
340 LET F(N,K) = D
345 NEXT K
346 NEXT N
350 FOR G = 45 TO 55
397 IF F(25,G) = F(24,G) THEN 400
398 LET X(25) = 1
399 GO TO 401
400 LET X(25) = 0
401 LET P = G
402 FOR J = 24 TO 2 STEP -1
404 LET V = J+1
405 IF X(V) = 1 THEN 436
430 LET M = 0
435 GO TO 441
436 LET M = A(V)
441 LET E = P-M
442 LET R = J-1
443 LET S = F(R,E)
445 IF F(J,E) = S THEN 460
450 LET X(J) = 1
451 GO TO 470
460 LET X(J) = 0
470 LET P = E
490 NEXT J
500 IF A(1) <= P THEN 520
```

```

505 LET X(1) = 0
512 G3 TJ 525
520 LET X(1) = 1
525 LET J = A(1)*X(1)
526 FOR C = 2 TJ 25
527 LET U = U + A(C)*X(C)
529 NEXT C
701 PRINT "USING" G " AS OUR TOTAL CONSTRAINT AN OPTICAL BENEFIT"
702 PRINT "WILL BE DERIVED FROM:"
705 FOR L = 1 TJ 8
706 LET W = 3*L - 2
707 LET T = 3*L - 1
708 LET Y = 3*L
709 PRINT "X(" G") =" X(0) "X(" T") =" X(T) "X(" Y") =" X(Y)
777 NEXT L
778 PRINT " X( 25 ) ="X(25)
788 PRINT " THIS OPTIMAL SELECTION USED" U " UNITS OF THE TOTAL"
789 PRINT "ALLOWABLE CONSTRAINT--"G", AND PRODUCED A BENEFIT"F(25,G)
790 PRINT " "
799 NEXT G
901 DATA 2,164.8,
902 DATA 4,142.2,
903 DATA 15,139.8,
904 DATA 6,127.3,
905 DATA 2,124.2,
906 DATA 14,127.3,
907 DATA 12,136.7,
908 DATA 6,128.9,
909 DATA 1,128.1,
910 DATA 2,117.2,
911 DATA 4,112.5,
912 DATA 3,149.2,
913 DATA 1,136.7,
914 DATA 2,133.6,
915 DATA 1,123.4,
916 DATA 3,132.0,
917 DATA 5,113.3,
918 DATA 3,115.6,
919 DATA 4,115.6,
920 DATA 6,111.7,
921 DATA 2,122.6,
922 DATA 3,119.5,
923 DATA 16,121.1,
924 DATA 2,135.2,
925 DATA 17,118.0,
999 END

```

III. CHECK Program

CHECK

```
25 DIM A(25)
30 DIM B(25)
35 DIM K(25)
40 FOR N = 1 TO 25
50 READ A(N)
51 READ B(N)
52 READ K(N)
55 NEXT N
56 LET U = A(1)*K(1)
57 LET V = B(1)*K(1)
58 FOR N = 2 TO 25
59 LET U = A(N)*K(N) + U
60 LET V = B(N)*K(N) + V
70 NEXT N
75 PRINT " THIS SELECTION USES" U "RESOURCE AND PRODUCES" V "BENEFIT."
901 DATA
999 END
```